Optimal Carbon Pricing of Closed-Loop Supply Chains: A Global-Optimisation of a Bi-Level Mixed-Integer Problem

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***Abstract*—We study the design of closed-loop supply chains under a regulator-imposed carbon tax, formulated as a bilevel mixed-integer program (BL-MIP). The regulator (leader) selects a discrete carbon-tax rate that minimises supply-chain emissions; the operator (follower) responds by optimising facility location, capacity, technology, transportation, and forward and reverse product flows to minimise cost inclusive of tax. We solve the resulting BL-MIP with a deterministic bounding procedure (DBP) that computes the regulator's optimum between two bounds: a linear-programming (LP) relaxation of the follower's problem provides a lower bound, and the follower's exact integer response provides an upper bound. The DBP iteratively tightens these two bounds until their gap is below a given tolerance. Three results emerge. First, emission abatement saturates at a network-specific threshold, beyond which higher taxes yield no further reduction. Second, abatement is governed by the technology menu — cleaner production and remanufacturing options — and expanding that menu can reduce both cost and emissions. Finally, expanding recovery capacity achieves the same abatement at a lower carbon price — each unit of recovery substitutes directly for a unit of taxation.**

***Index Terms*—bilevel optimization, carbon pricing, closed-loop supply chain network design, mixed-integer programming, environmental regulation**

I. Introduction

Carbon taxation prices each unit of CO_2-equivalent emitted, producing emission cuts when well calibrated [1], and closed-loop architectures that integrate recovery and remanufacturing enlarge the available abatement levers [2]. Crucially, the regulator-operator interaction is hierarchical: the regulator commits to a tax before the operator responds, so a single-level model that treats the tax as fixed cannot represent the regulator's problem of choosing a rate that induces a desirable response.

We model this as a bilevel program [3]: the leader sets the carbon tax to minimize total emissions and, among rates achieving that minimum, levies the lowest one; the follower solves a multi-echelon, multi-period design problem with integer facility, technology, and transport decisions, minimizing cost inclusive of tax. The resulting bilevel mixed-integer program (BL-MIP) is NP-hard [4].

This paper makes three contributions: (i) a BL-MIP with a discrete carbon tax decision with a closed-loop supply-chain design problem; (ii) a deterministic bounding procedure (DBP) converging to a ϵ-optimal solution; and (iii) computational results show that emission abatement saturates at a network-specific tax threshold, that technology is the binding lever below that threshold, and that expanding recovery capacity can substitute directly for taxation. Section II states the model, Section III the DBP, and Section IV the results.

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II. Bilevel Model

The closed-loop network includes suppliers, factories, warehouses, air/sea hub terminals, and markets, linked by forward flows of raw materials and finished products and by reverse flows of end-of-life items recovered for remanufacturing. Over a multi-period planning horizon, the follower decides the network structure — facility, technology, transport, and fleet choices — and production, remanufacturing, inventory, and flow quantities.

Let $\mathbf{z} \in \mathcal{X}$ denote the follower's decision vector, where $\mathcal{X}$ is the follower's feasible region induced by (i) flow and inventory balance, (ii) reverse-logistics and returns, (iii) raw-material supply, facility-opening, (iv) area-, inventory and fleet-capacity, (v) technology-, and transport-selection, and (vi) intermodal hub routing. The regulator's decision variable is the carbon-tax rate τ (USD/tCO_2e), subject to integer values in $\mathcal{T} = \{\tau_{\min}, \tau_{\min}+1, \ldots, \tau_{\max}\}$. Denoting the follower's cost and emissions by $\mathrm{Cost}(\mathbf{z})$ and $\mathrm{Emis}(\mathbf{z})$, its optimal response to a given rate τ is the correspondence

$$\mathcal{S}(\tau) \;\triangleq\; \operatorname*{arg\,min}_{\mathbf{z}\in\mathcal{X}} \big\{\mathrm{Cost}(\mathbf{z}) + \tau\,\mathrm{Emis}(\mathbf{z})\big\}. \tag{1}$$

Anticipating this response, the leader minimizes resulting emissions and, among rates attaining that minimum, the tax itself (avoiding unnecessary taxation). This defines the bilevel program

$$\text{(BL)} \quad \operatorname*{lexmin}_{\tau,\,\mathbf{z}} \quad \big(\mathrm{Emis}(\mathbf{z}),\ \tau\big) \tag{2a}$$

$$\text{s.t.} \quad \tau \in \mathcal{T}, \qquad \mathbf{z} \in \mathcal{S}(\tau). \tag{2b}$$

Since the follower's problem (1) is itself a mixed-integer program for every fixed τ, problem (2) is at least as hard, establishing the NP-hardness of the BL-MIP noted in Section I [4].

III. Solution Method

We solve (2) with a *deterministic bounding procedure* (DBP), adapted from [5], which generates a sequence of converging lower and upper bounds on the leader's optimal emissions, in the following steps.

Step 1 (initialization). Set $\mathrm{LB} \leftarrow -\infty$. Solve the *relaxation* of (2) obtained by dropping the coupling constraint $\mathbf{z} \in \mathcal{S}(\tau)$, i.e., $\min\{\mathrm{Emis}(\mathbf{z}) : \mathbf{z} \in \mathcal{X},\ \tau \in \mathcal{T}\}$, to obtain a first candidate rate τ.

Step 2 (follower response). Solve the follower's exact problem (1) at τ to obtain its true optimum $\nu(\tau)$.

Step 3 (optimistic tie-break). Since $\mathcal{S}(\tau)$ may contain several optimal solutions, solve $\min\{\mathrm{Emis}(\mathbf{z}) : \mathbf{z} \in \mathcal{X},\ \mathrm{Cost}(\mathbf{z}) + \tau\,\mathrm{Emis}(\mathbf{z}) \le \nu(\tau)\}$ to recover its lowest-emissions member $\mathbf{z}^*$.

Step 4 (upper bound). The pair $(\tau, \mathbf{z}^*)$ is feasible for (2). If $\mathrm{Emis}(\mathbf{z}^*)$ improves on the incumbent, update $\mathrm{UB} \leftarrow \mathrm{Emis}(\mathbf{z}^*)$ and $\tau^\dagger \leftarrow \tau$.

Step 5 (tightening). Add to the relaxation of Step 1 the cut $\mathrm{Cost}(\mathbf{z}) + \tau\,\mathrm{Emis}(\mathbf{z}) \le \mathrm{Cost}(\mathbf{z}^*) + \tau\,\mathrm{Emis}(\mathbf{z}^*)$ and the bound $\mathrm{Emis}(\mathbf{z}) \le \mathrm{UB}$, ruling out τ from being proposed again; re-solve it for a new candidate τ and update LB.

Step 6 (stopping test). Stop and return $\tau^\dagger$ if $(\mathrm{UB} - \mathrm{LB})/\mathrm{UB} \le \epsilon$, or if neither bound has improved over 10 consecutive iterations; otherwise return to Step 2.

IV. Preliminary Results

We generated eight instances, following the instance-generation guidelines of [6]: network (17/25 nodes), product (4/8 types), and horizon (3/6 periods). All 8 instances converge with a relative gap no greater than $\le 1.36 \times 10^{-5}$ in 2–4 DBP iterations (Table I). We also run two further experiments. A *scenario analysis* re-solves each base instance under two menu expansions – technology (3→6 production and 3→6 remanufacturing technologies) and transport (6→12 modes, two additional road/air/sea options each) – for 16 runs. A *sensitivity analysis* then perturbs each base instance one factor at a time (OAT) across 11 model factors, grouped as: demand; production and transportation capacity; warehouse and air/sea hub cost; return availability; return cost; remanufacturing bill-of-materials; production and upstream supply emissions; and carbon-tax bounds – for 152 runs.

Four findings emerge from our runs.

First, carbon pricing reduces emissions by 8.6–57.6% at a cost increase of 3.4–75.9%, and the two are not consistently related: I25 M4 T6 achieves the largest cut (-57.6%) at the lowest abatement cost ($3.62/t), whereas I17 M8 T3 and I25 M4 T3 incur a cost increase above 75% for a comparatively modest 24–35% reduction.

TABLE I
DBP solution per base instance.

Instance	$\tau^\star$	ΔE (%)	ΔC (%)
I17 M4 T3	200	-28.4	$+14.2$
I17 M4 T6	190	-9.3	$+3.4$
I17 M8 T3	140	-23.9	$+75.6$
I17 M8 T6	200	-8.7	$+5.0$
I25 M4 T3	180	-35.4	$+75.9$
I25 M4 T6	20	-57.6	$+20.6$
I25 M8 T3	140	-48.4	$+14.9$
I25 M8 T6	100	-8.6	$+5.2$

$\tau^\star$ in USD/tCO_2e; ΔE and ΔC are the percentage changes in emissions and cost, respectively, relative to each instance's $\tau = 0$ baseline.

Second, because the operator chooses among finitely many configurations, its emission response saturates beyond a threshold: widening the ceiling from $200/t to $800/t moves I25 M8 T3's optimum eighteen-fold ($20→$360/t) for a further 0.04% cut, while I17 M8 T3 stays flat across $140–$310/t – risking arbitrarily high taxes for no added abatement.

Third, technology, is the binding lever: widening the technology menu (3→6/family) cuts emissions in all 8 instances (-18.9% to -37.8%) and, on I25 M4 T3, cost by 41.8% – a Pareto gain regardless of $\tau^\star$. Widening transport modes (6→12) instead leaves emissions near-unchanged (0.24%).

Fourth, recovery capacity substitutes for carbon pricing: greater return availability lowers the tax rate needed to reach a given emissions level (elasticity -0.744) without changing that level itself (elasticity -0.001). More recoverable material lets the cheaper remanufacturing pathway absorb output with less tax pressure – recovery substitutes for, rather than competes with, taxation.

V. Conclusion and Ongoing Work

We are also solving a multi-objective formulation in which the carbon tax is fixed and cost is traded off against emissions, to quantify the gain from the bilevel program.

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